# MTH241 Fall 2024: Quiz 10

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Closed book, no calculator, show your work clearly.

1. (5pt) Let $\Sigma$ be the surface of the outward-oriented cube with opposite corners $(0,0,0)$ and $(2,2,2)$ and $\vec{F}(x,y,z) = 5x\vec{i} + 2y\vec{j} - 2z\vec{k}$. Evaluate the surface integral $\iint_\Sigma \vec{F}\cdot\vec{n}dS$. (Hint: use the Divergence Theorem)

$$\iint_\Sigma F\cdot n\,ds = \iiint_D \operatorname{div}(F)\,dV = \iiint_D 5+2-2\,dV = \iiint_D 5\,dV = 5\cdot \text{volume of cube} = 5\cdot 2^3 = 40$$

2. (5pt) Let $\Sigma$ be the part of the sphere $x^2+y^2+z^2 = 4$ which is above the xy-plane with the unit normal pointing outward. Compute $\iint_\Sigma(\operatorname{curl}\vec{F})\cdot\vec{n}\,dS$ where $\vec{F}(x,y,z) = y\vec{i} + z\vec{j} + x\vec{k}$. (Hint: Use the Stokes's Theorem)

1st way: $\iint_\Sigma \operatorname{curl}(F)\cdot n\,ds = \int_C F\cdot dr$ $\quad r(t) = \begin{bmatrix} 2\cos(t) \\ 2\sin(t) \\ 0 \end{bmatrix} \rightsquigarrow \frac{dr}{dt} = \begin{bmatrix} -2\sin(t) \\ 2\cos(t) \\ 0 \end{bmatrix}$ counterclock

$\Sigma$

$C$

comp orient

$$\text{So } \iint_\Sigma \operatorname{curl}(F)\cdot n\,ds = +\int_C F\cdot dr = \int_0^{2\pi} F(r(t))\cdot \frac{dr}{dt}\,dt = \int_0^{2\pi} \begin{bmatrix} 2\sin(t) \\ 0 \\ 2\cos(t) \end{bmatrix}\cdot\begin{bmatrix} -2\sin(t) \\ 2\cos(t) \\ 0 \end{bmatrix}$$

$$= \int_0^{2\pi} -4\sin^2(t) = -4\int_0^{2\pi}\frac{1-\cos(2t)}{2} = -4\pi$$

2nd way:

$\Sigma_1 \rightsquigarrow \Sigma_2$

$$\iint_{\Sigma_1} \operatorname{curl}(F)\cdot n\,ds = \iint_{\Sigma_2} \operatorname{curl}(F)\cdot n\,ds \qquad \Sigma_2 : r(r,\theta) = \begin{bmatrix} r\cos(\theta) \\ r\sin(\theta) \\ 0 \end{bmatrix}$$

$$r_r \times r_\theta = \begin{bmatrix} 0 \\ 0 \\ r \end{bmatrix}$$

$$\operatorname{curl}(F) = \begin{vmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ y & z & x \end{vmatrix} = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix} \quad \text{so } \iint_{\Sigma_2} \operatorname{curl}(F)\cdot n\,ds = \int_0^{2\pi}\int_0^2 \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}\cdot\begin{bmatrix} 0 \\ 0 \\ r \end{bmatrix} dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^2 -r\,dr\,d\theta = -4\pi$$

**Second page:**